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Simulating the Complex Behavior of a Leaky Faucet

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The complex behavior of leaky faucets is obtained by numerical simulation using a stochastic method introduced by Manna *et al.* and the results are compared with experimental data. Typical return maps of thin faucets are reproduced.

KEY WORDS: Dynamical complex systems; chaos; lattice simulation.

Many experiments[(1-6)] have confirmed that the dripping faucet presents complex behavior, e.g., chaos, intermittency, and hysteresis. Although the deterministic equations governing this system are not known, a mathematical model supposing that the forming drop oscillates was proposed by Martien *et al.*[(1)] The equation describing this motion is the same as that of the mass–spring problem:

$$\frac{d}{dt}\left(m\frac{dy}{dt}\right)=mg-ky-b\frac{dy}{dt} \tag{1}$$

where m is the mass of the forming drop, y is its position and g, k, and b are constant parameters. The mass increases linearly with time, while the oscillation frequency decreases. At a critical moment that depends on the mass, the drop breaks away, setting the initial conditions for the next drop. Some solutions of this equation for several values of parameters reproduce in a qualitative sense some experimental return maps taken from "large" faucets (>1 mm width).

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On the other hand, Manna *et al.*[7] proposed a stochastic method to determine the shape of a drop on a wall. This is an optimization problem, since the shape must minimize the surface tension for a given volume. Manna et *al.* found a transition between hanging and sliding phases and determined some critical exponents. In the present work we adapt this method to simulate the formation of a drop in a leaky faucet in order to investigate the dynamic behavior of this system as many drops fall successively.

As in ref. 7 we work on a square lattice and consider Ising variables $\sigma_i = \pm 1$, where $\sigma = +1$ represents the fluid and $\sigma = -1$ represents the air. The Hamiltonian is still

$$\mathscr{H} = -\sum_{\mathrm{nn}} \sigma_i \sigma_j - \sum_{\mathrm{nnn}} \sigma_i \sigma_j - \sum_j h_j \sum_{\mathrm{row}\, j} \sigma_i \tag{2}$$

where the first sum is over the nearest neighbors and the second one is over the next nearest neighbors. Both simulate the effects of molecular attraction and surface tension. The third summation refers to the presence of gravity, and h_j has the same value for all sites in the jth row, counted from above to below. It is defined as $h_j = gj$, where the "gravity" g is a constant and $j = 1, 2,..., L$, where L is the vertical lattice size. We impose mass conservation by reversing always two nonneighboring spins (neither nn or nnn), one up and the other down, simultaneously. They are randomly chosen, one from the current drop internal boundary $(+1)$ and the other from the external boundary (-1), in order to avoid bubbles inside the drop. Temperature is introduced through the Metropolis algorithm. The boundary conditions are: W neighboring spins will be set to $+1$ at the center of an additional row $j = 0$ which remains fixed during all the dynamic process. This condition models the injection of fluid, as we will describe below. W is the internal width of the faucet and it is a relevant parameter, as pointed out by Dreyer and Hickey.[4] The time evolution is as follows:

(a) At each integer time step $t = 0, 1, 2$ we lower the drop one row down without modifying its shape by taking $\sigma_i^j = \sigma_i^{j-1}$ for rows $j = L, L-1,..., 1$. This is the injection of fluid at a constant rate taken in discrete time steps.

(b) Between two successive fluid injections at times t and $t+1$ we allow each point of the current drop internal boundary to relax a fixed number N of times through the pair update dynamics already quoted. This procedure is adopted in order to synchronize the two independent time scales relevant to the physical problem: the rate of fluid injection and the characteristic relaxation time of the drop shape. Accordingly, immediately after each fluid injection at time t, we measure the current drop internal

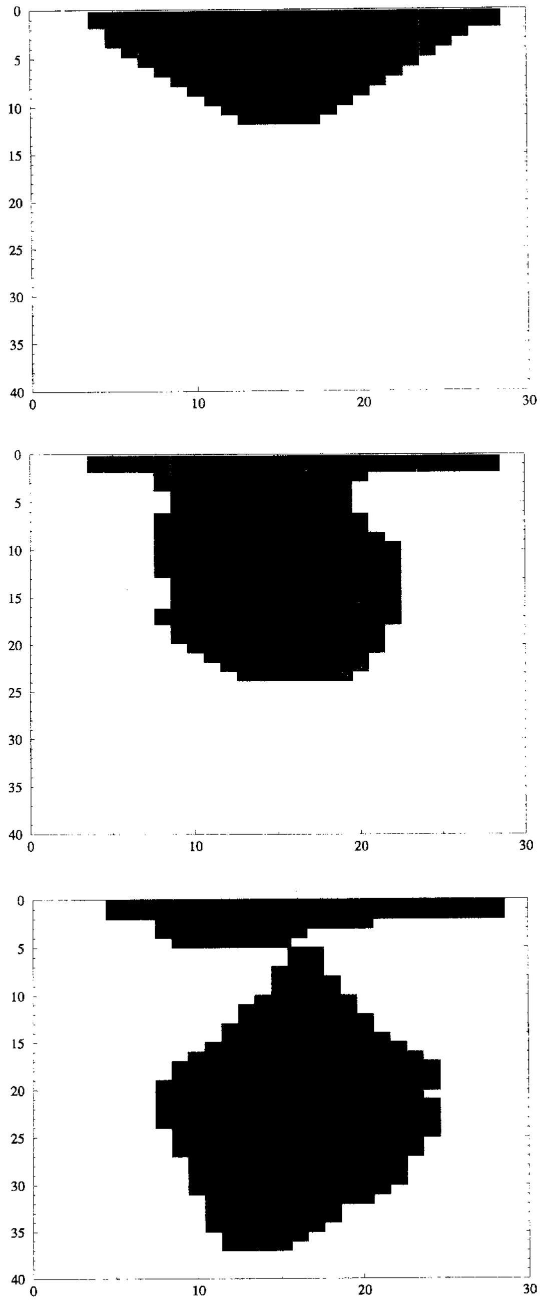


Fig. 1. Three stages of drop formation.

perimeter p_t and define the number $n_t = Np_t$ of Kawasaki trials (pair updates) to be performed until the next fluid injection at time $t+1$.

(c) After each pair update we tested if the drop is disconnected from the top, by performing a burning algorithm[8] from above. The check for disconnection is performed taking into account only nearest neighbors. If the disconnection occurs at the kth pair update, the total time since the last drop fall is $T = t + k/n_t$. Following the experimenters, we store a series of many successive T's in the computer memory for later analysis. The disconnected part is discarded and the process goes on with a newly forming drop, starting from the current part still connected to the faucet.

In Fig. 1, we show three stages of drop dynamics: at the beginning, the neck formation, and the drip, where we can note the asymmetry in the drop. The shape of a drop can be smoothed by averaging over many configurations, but in this work we are interested only in the time interval between two successive drops. Since we do not include kinetic terms, the drop cannot oscillate. In Fig. 2, we present some return maps plotting each interval time (x axis) against the next interval time (y axis) for different values of g. In this case, we use $W = 20$ and $N = 100$. The patterns are (Fig. 2e) period-1, (b) intermittency-3, (a) intermittency-4, (d) L-shaped, and (c, f, g) other strange attractors. Unlike other authors, we adopt the term "intermittency-n" instead of "period-n" for patterns like Figs. 2a and 2b. In these cases, the system follows *finite* periodic sequences, but the successive lengths of these sequences do not present a periodic behavior. Patterns like these were reported by Dreyer and Hickey[4] for small faucets (inner diameter of 0.47 mm). Patterns like Fig. 2i, intermittency-9, were not reported in experiments. We conjecture that this kind of attractor might be either not resolved in the experiment because it should appear in higher flow rates (close to the laminar regime) or it was a transient effect in simulations.

We believe that oscillations (that are not present in our model) are not responsible for the complex behavior of leaky faucets, at least for small faucets as reported in ref. 4, in opposition to Martien *et al.*'s assumption. The dimension of the space seems to be not relevant, since even 2D drop as simulated here also presents complex behavior, although the formation of necks follows from a completely different topology (in 3D, one has two independent curvatures at the neck). In order to reproduce the attractors of larger faucets, we should simulate larger lattices. However, for lower flow rates ($g = 0.2$ for $W = 20$ and $N = 100$), it takes one drop/hr in the Sun SPARC 2, which keeps us from performing simulations for much larger faucets. More details on the simulations are presented in ref. 9, with the use of multispin coding.[10] This procedure can be extended to study

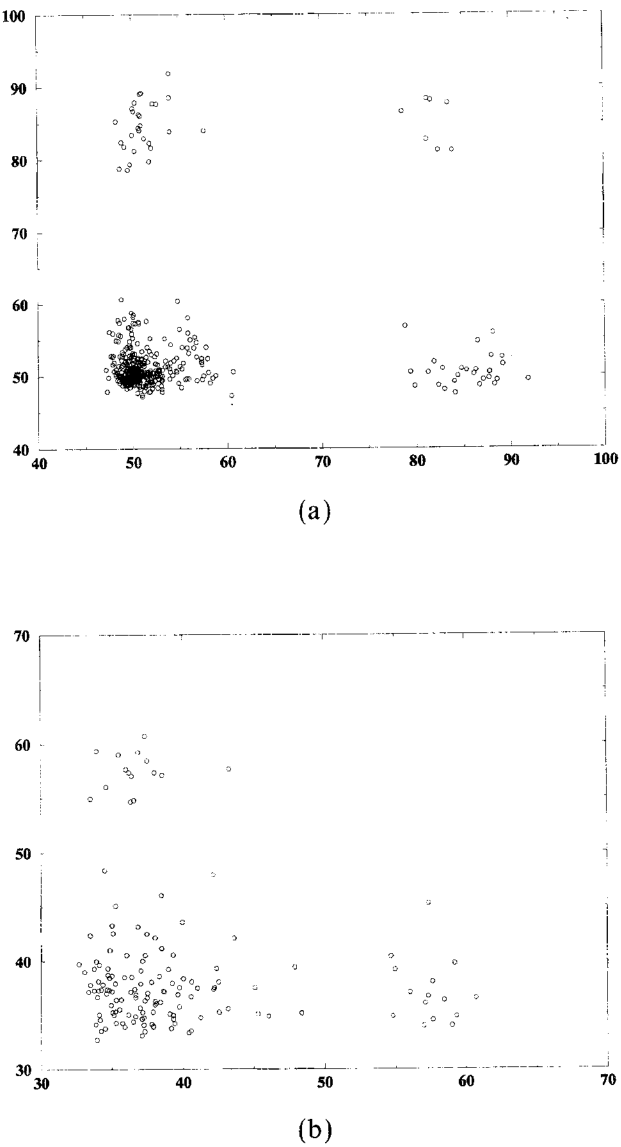
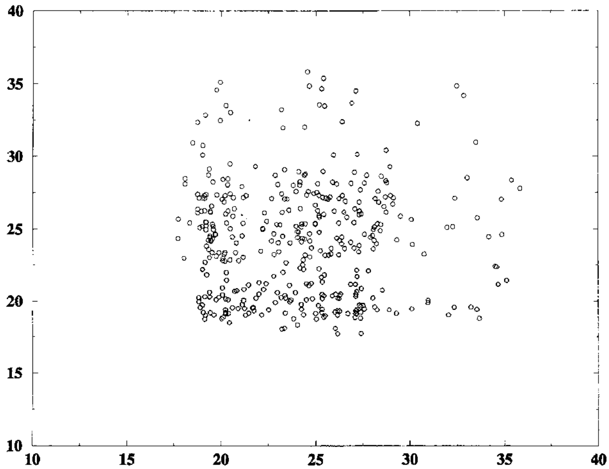
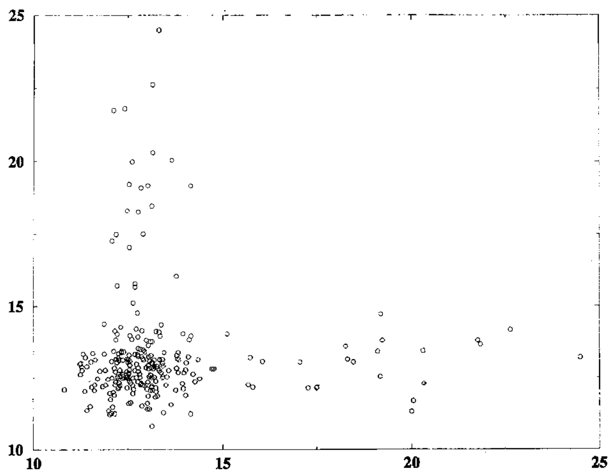


Fig. 2. Return maps for $W=20$, $N=100$, and the following values of g: (a) 0.4, (b) 0.6, (c) 1, (d) 1.5, (e) 2.5, (f) 4, (g) 7, (h) 8, and (i) 15.

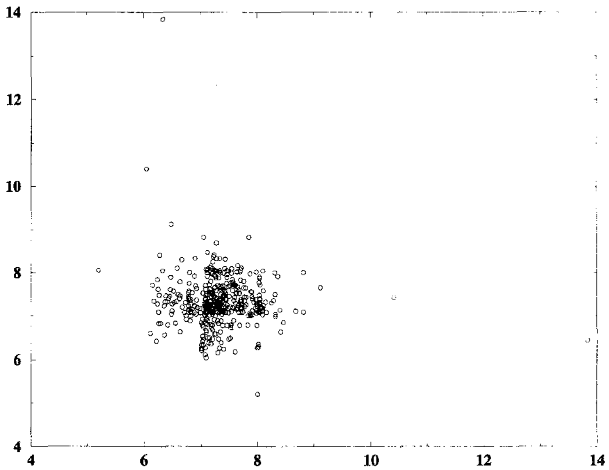


(c)

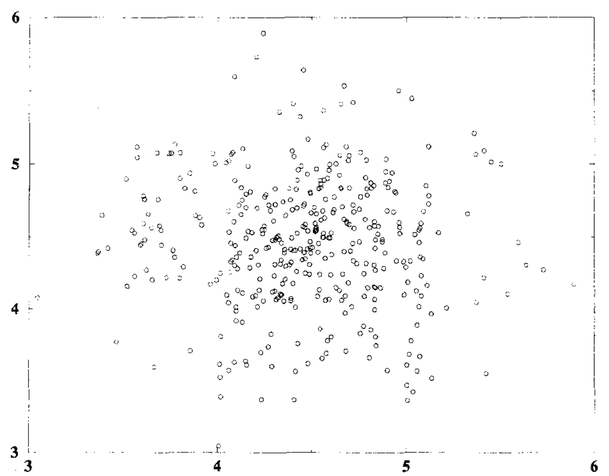


(d)

Fig. 2 (*continued*)

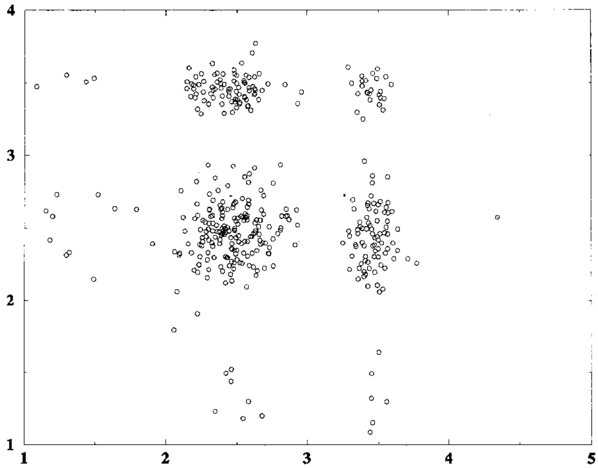


(e)

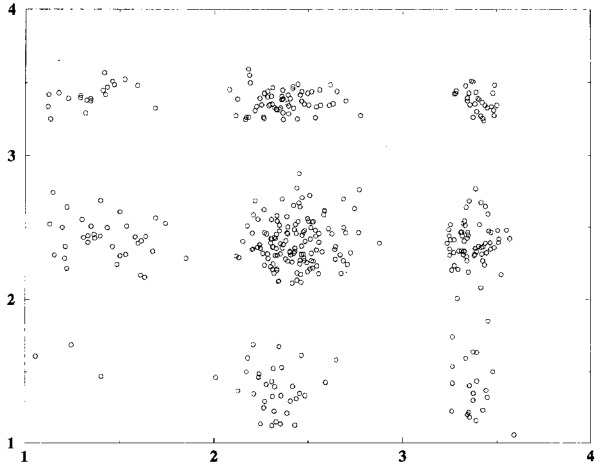


(f)

Fig. 2 (*continued*)

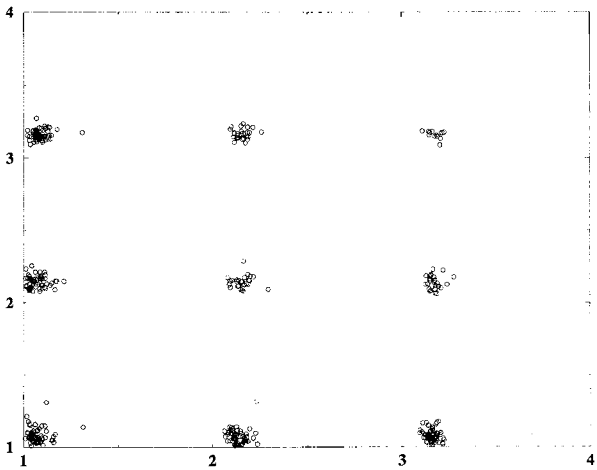


(g)



(h)

Fig. 2 (*continued*)



(i)

Fig. 2 (*continued*)

temperature[9] and surface tension effects,[3] size of the of faucet,[4,9] crisis,[5] and hysteresis.

A possible interpretation for patterns like those in Fig. 2 is the following. Nearly stationary surface waves favor the formation of an integer number of necks along the drop height for certain combinations of the parameters relevant to the problem (fluid injection rate, viscosity, gravity, etc.). For combinations such that only one neck appears, the periodic behavior is observed, whereas two necks correspond to intermittency-3 or -4 and three necks correspond to intermittency-9. The difference between Figs. 2b and 2a is that only one large drop follows a sequence of small ones in Fig. 2b, whereas Fig. 2a also alternates sequences of large and small drops. According to this interpretation, the remainder of nondiscrete patterns (Figs. 2d and 2f) correspond to nonstationary surface waves, whereas Figs. 2c, 2g, and 2h correspond to an intermediate situation. For large faucets the integer number of necks would be large enough such that the return maps should seem continuous, as corroborated by experiments.

Concluding, we have performed numerical simulations of drops falling in sequence from a leaky faucet. We adopt an Ising-like model with adequately designed dynamics, measuring the time series of successive falling drops. Our results compare very well with the experimental data available for tiny faucets (≈ 0.47 mm).[4]

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